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Theory of strong-electromagnetic-wave propagation in an electron-positron-ion plasma

V. I. Berezhiani,* M. Y. El-Ashry,† and U. A. Mofiz‡
International Centre for Theoretical Physics, Trieste, Italy
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The propagation of intense electromagnetic radiation in an admixture of unmagnetized electron-positron-ion plasma is investigated analytically. It is shown that relativistically intense electromagnetic radiation in the presence of heavy ions, in contrast to the case of a pure electron-positron plasma, may be localized with the generation of a humped bipolar potential in the plasma. This potential may cause an acceleration of particles in such a plasma.

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Recently, the nonlinear propagation of electromagnetic (EM) waves in electron-positron plasma has attracted the interest of researchers [1] due to the fact that the electron-positron plasmas are found near the polar cap of a pulsar, in the active galactic nuclei, as well as in the early universe. The process of electron-positron pair creation and annihilation occurs in relativistic plasma at high temperatures, when the temperature of the plasma exceeds the rest mass of electrons [2]. In [3] it was shown that the positrons can be used to probe the particle transport in tokamak plasma. Electron-positron pair production can also be possible during intense short laser pulse propagation in plasma [4]. Because of the sufficient lifetime of the positrons, the plasma becomes an admixture of electrons, positrons, and ions. Such a three-component plasma can indeed be created in laboratory plasma [5], and has been studied in different models of pulsar magnetospheres [6]. The importance of the three-component admixture plasma has led to much related theoretical investigations. Of these, Rizzato [7] investigated the weakly nonlinear localization of obliquely modulated high-frequency EM waves, and found that the amplitude of the wave turns out to be a strongly dependent function of the angle between the slow modulation and the fast spatial variation. Berezhiani, Tsintsadze, and Shukla [8] studied the wake field generation by short, intense EM wave packets to show that such a plasma can drastically reduce the amplitude and wavelength of the generated electrostatic wake field. In the present paper, the propagation of the strong EM waves in an electron-positron and ion cold unmagnetized plasma admixture is considered, aiming to find analytically the localized soliton-type solution.

To describe the admixture of plasma made up of electrons, positrons, and ions, we use Maxwell equations, in which the fields are expressed in terms of the potentials, i.e.,

$$\mathbf{E}=-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}-\nabla\varphi\ , \quad \mathbf{B}=\nabla\times\mathbf{A}\ , \tag{1}$$

where the Coulomb gauge $\nabla\cdot\mathbf{A}=0$ is fulfilled. Accordingly, using Poisson's equation, one may obtain the following equations for the potentials:

$$\frac{\partial^2 \mathbf{A}}{\partial t^2}-c^2\nabla\mathbf{A}=4\pi c\mathbf{J}-c\frac{\partial}{\partial t}(\nabla\varphi) \tag{2}$$

and

$$\Delta\varphi=-4\pi\rho\ . \tag{3}$$

Here, ρ and $\mathbf{J}$ are the charge and current densities given by

$$\rho=\sum_\alpha n_\alpha q_\alpha\ , \quad \mathbf{J}=\sum_\alpha n_\alpha q_\alpha \mathbf{v}_\alpha\ , \tag{4}$$

where α indicates the particle species α ($=e, p, i$ for electrons, positrons, and ions, respectively); q_α and n_α are the charge and the density of the corresponding particle α. We shall consider the case in which the admixture equilibrium state is characterized by $n_{0e}=n_{0p}+n_{0i}$, where $n_{0\alpha}$ is the equilibrium density of the particle α.

The relativistic equations of motion of different particles of the unmagnetized plasma admixture is written as [9]

$$\frac{\partial}{\partial t}\mathbf{P}_\alpha+m_\alpha c^2\nabla\gamma_\alpha=q_\alpha\left[-\nabla\varphi-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\right]\ , \tag{5}$$

where

$$\mathbf{P}_\alpha=m_\alpha\gamma_\alpha\mathbf{v}_\alpha\ ,$$
$$\gamma_\alpha=\left[1-\frac{v_\alpha^2}{c^2}\right]^{-1/2}=\left[1+\frac{P_\alpha^2}{m_\alpha^2c^2}\right]^{1/2}\ , \tag{6}$$

and m_α is the rest mass of the particle α.

The continuity equation for the particle α is

$$\frac{\partial n_\alpha}{\partial t}+\nabla\cdot(n_\alpha\mathbf{v}_\alpha)=0\ . \tag{7}$$

*Permanent address: Institute of Physics, The Georgian Academy of Science, Tbilisi 380077, The Republic of Georgia.

†Permanent address: Department of Physics, Faculty of Science, Suez Canal University, Ismailia, Egypt.

‡Permanent address: Institute of Nuclear Science and Technology, Atomic Energy Research Establishment, Savar, G.P.O. Box 3787, Dhaka, Bangladesh.

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We are looking for a localized one-dimensional solution of this system of equations for a circularly polarized EM wave, where the vector potential **A** can be expressed as

$$\mathbf{A}=\tfrac{1}{2}(\hat{\mathbf{x}}+i\hat{\mathbf{y}})A(\xi)\exp[i(kz-\omega t)]+\text{c.c.} \tag{8}$$

Here $\xi=z-Vt$, and $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ are the unit vectors.

Analyzing separately the longitudinal (parallel to the direction of the wave propagation) and transverse (perpendicular plane) parts of the equation of motion and the wave equations, the transverse part of the equation of motion is immediately integrated to give

$$\mathbf{P}_{\alpha\perp}=-\frac{q_\alpha}{c}\mathbf{A}\ , \tag{9}$$

where the constant of integration is set equal to zero, since the particles were assumed to be immobile at infinity where the field is zero.

Meanwhile, the longitudinal part of the equation of motion takes the following form:

$$\frac{\partial P_{\alpha z}}{\partial t}+m_\alpha c^2\frac{\partial\gamma_\alpha}{\partial z}=-q_\alpha\frac{\partial\varphi}{\partial z}\ , \tag{10}$$

with

$$\gamma_\alpha=\left[1+\frac{e^2|A|^2}{m_\alpha^2c^2}+\frac{P_{\alpha z}^2}{m_\alpha^2c^2}\right]^{1/2}\ , \tag{11}$$

and, accordingly, Eqs. (2), (3), and (7) may be rewritten as

$$\frac{\partial^2\mathbf{A}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}=\frac{4\pi e^2}{m_e^2c^2}\left[\frac{n_e}{\gamma_e}+\frac{n_p}{\gamma_p}+\mu\frac{n_i}{\gamma_i}\right]\mathbf{A}\ , \tag{12}$$

$$\frac{\partial^2\varphi}{\partial z^2}=4\pi e(n_e-n_p-n_i)\ , \tag{13}$$

$$\frac{\partial n_\alpha}{\partial t}+\frac{\partial}{\partial z}\left[\frac{n_\alpha P_{\alpha z}}{m_\alpha\gamma_\alpha}\right]=0\ , \tag{14}$$

where $\mu=(m_e/m_i)$ is the electron to ion mass ratio. Since ions are much heavier than the electrons, i.e., $\mu\ll 1$, then for simplicity the ion motion will be neglected in our present purpose of study. The subscript α, henceforth, will indicate the electrons and the positrons only.

It is now convenient to introduce the following dimensionless quantities:

$$P_\alpha=\frac{P_\alpha}{m_ec}\ ,\quad \sigma=\frac{eA}{m_ec^2}\ ,\quad \Phi=\frac{e\varphi}{m_ec^2}\ ,$$

$$\frac{n_\alpha}{n_{0\alpha}}=N_\alpha\ ,\quad \eta=\frac{\omega_e}{c}\xi\ ,$$

where $\omega_e=(4\pi n_{0e}e^2/m_e)^{1/2}$ is the electron Langmuir frequency.

Due to the fact that A depends only on the variable η, we can integrate Eq. (10) (in the same manner as in [10,11]), to get

$$P_{\alpha z}=\frac{V_0}{R}\left\{\left[1-\frac{q_\alpha}{|q_\alpha|}\Phi\right]-\left[\left[1-\frac{q_\alpha}{|q_\alpha|}\Phi\right]^2-R(1+\sigma^2)\right]^{1/2}\right\}\ , \tag{15}$$

$$\gamma_\alpha=\frac{1}{R}\left\{\left[1-\frac{q_\alpha}{|q_\alpha|}\Phi\right]-V_0\left[\left[1-\frac{q_\alpha}{|q_\alpha|}\Phi\right]^2-R(1+\sigma^2)\right]^{1/2}\right\}\ , \tag{16}$$

$$N_\alpha=\gamma_\alpha V_0\left[\left[1-\frac{q_\alpha}{|q_\alpha|}\Phi\right]^2-R(1+\sigma^2)\right]^{-1/2}\ , \tag{17}$$

where $V_0=(V/c)$ and $R=1-V_0^2$.

Using (8) and (15)–(17), Eqs. (12) and (13) could then be rewritten as

$$\sigma\frac{d^2\sigma}{d\eta^2}=-\Omega\Psi+\frac{V_0}{R}\Psi\left\{\frac{1}{[(1+\Phi)^2-R(1+\Psi)]^{1/2}}+\frac{\epsilon}{[(1-\Phi)^2-R(1+\Psi)]^{1/2}}\right\}=G(\Phi,\Psi)\ , \tag{18}$$

$$\frac{d^2\Phi}{d\eta^2}=\frac{V_0}{R}\left\{\frac{1+\Phi}{[(1+\Phi)^2-R(1+\Psi)]^{1/2}}-\frac{\epsilon(1-\Phi)}{[(1-\Phi)^2-R(1+\Psi)]^{1/2}}-\frac{1-\epsilon}{V_0}\right\}=F(\Phi,\Psi)\ , \tag{19}$$

where $\Psi=\sigma^2$, $\epsilon=(n_{0p}/n_{0e})$ $(0\le\epsilon\le 1)$ and $\Omega=(\omega/\omega_e)^2$, and we assume that $V=kc^2/\omega$, which determines only the most successful choice of the coordinate system in which the equation for the envelope takes the simple form [10].

In the case of pure electron-positron plasma (i.e., when $\epsilon=1$) Eq. (19) reduces to Eq. (23) of Ref. [12]. For our boundary conditions, it is shown [12] that the only possible solution is $\Phi=0$. The potential vanishes because of the same radiative pressure for electrons and positrons due to their equal masses. Under these conditions Eq. (18) [the only one now remaining from the system of equations (18) and (19)] does not have a soliton solution. In our case, due to the presence of ions the radiative pressure creates a finite potential. Note that the case of electron-ion plasma ($\epsilon=0$) has been considered in [11].

This system has an "integral of motion" and it is given by

$$\left[\frac{d\sigma}{d\eta}\right]^2-\frac{1}{R}\left[\frac{d\Phi}{d\eta}\right]^2=-\Omega\Psi-\frac{2V_0}{R^2}[(1+\Phi)^2-R(1+\Psi)]^{1/2}-\frac{2V_0}{R^2}\epsilon[(1-\Phi)^2-R(1+\Psi)]^{1/2}+\frac{2(1-\epsilon)}{R^2}\Phi+E=H(\Phi,\Psi)\ , \tag{20}$$

where the constant of integration can be calculated from the boundary condition of the localized solution, and is found to be

$$E=\frac{2V_0^2}{R^2}(1+\epsilon) . \tag{21}$$

We have introduced the $G(\Phi,\Psi)$, $F(\Phi,\Psi)$, and $H(\Phi,\Psi)$ to find some analytical solution in accordance with an early investigation [11].

Making use of the "energy integral" (20), it is possible to eliminate the independent variable η between (18) and (19), yielding the following equation for Φ in terms of Ψ alone:

$$\begin{aligned}4\Psi H(\Phi,\Psi)\frac{d^2\Phi}{d\Psi^2}-\frac{8}{R}G(\Phi,\Psi)\Psi\left[\frac{d\Phi}{d\Psi}\right]^3\\+\frac{4}{R}\Psi F(\Phi,\Psi)\left[\frac{d\Phi}{d\Psi}\right]^2\\+2[G(\Phi,\Psi)+H(\Phi,\Psi)]\frac{d\Phi}{d\Psi}-F(\Phi,\Psi)=0 .\end{aligned} \tag{22}$$

We assume that the arising electrostatic field Φ is the function of the field amplitude Ψ. So we introduce the series expansions

$$\begin{aligned}&\Phi(\Psi)=\sum_n c_n\Psi^n ,\\&(1-\Phi)^2-R(1+\Psi)=\sum_n a_n\Psi^n ,\\&(1+\Phi)^2-R(1+\Psi)=\sum_n b_n\Psi^n ,\end{aligned} \tag{23}$$

with $c_0=0$, $a_0=b_0=1-R=V_0^2$ from the boundary condition and, of course, the coefficients a_n, b_n can be expressed in terms of c_n's for $n=1,2,3,\ldots$. We also assume the validity of the expansions

$$\begin{aligned}&[(1-\Phi)^2-R(1+\Psi)]^{\pm(1/2)}=V_0^{\pm1}\left[1\pm\frac{1}{2V_0^2}\sum_n a_n\Psi^n\right] ,\\&[(1+\Phi)^2-R(1+\Psi)]^{\pm(1/2)}=V_0^{\pm1}\left[1\pm\frac{1}{2V_0^2}\sum_n b_n\Psi^n\right] ,\end{aligned} \tag{24}$$

to be checked with the final solution.

Making use of the expansions (23) and (24) in (22), a power series in Ψ is obtained, and then equating the coefficients of the polynomial to zero, one can easily determine the coefficients c_n in terms of the free parameters Ω, V_0, and ϵ. This is done in the Appendix, where the series is found to be rapidly convergent for certain values of the parameters.

For $G(\Phi(\Psi),\Psi)\equiv G(\Psi)$, Eq. (18) becomes

$$2\Psi\left[\frac{d^2\Psi}{d\eta^2}\right]-\left[\frac{d\Psi}{d\eta}\right]^2-4\Psi G(\Psi)=0 , \tag{25}$$

with a first integral

$$\frac{1}{\Psi}\left[\frac{d\Psi}{d\eta}\right]^2-4\int\frac{G(\Psi)}{\Psi}d\Psi=\text{const}=0 , \tag{26}$$

while the last equation follows from the boundary conditions of the localized solution. Expanding the function $G(\Psi)$, defined in (18) according to (23) and (24), Eq. (26) becomes

$$\left[V_0\left[\frac{d\Psi}{d\eta}\right]\right]^2=4V_0^2\left\{\frac{1}{R}(1+\epsilon)-\Omega\right\}\Psi^2-\frac{2}{R}\sum_{n=1}\frac{b_n+\epsilon a_n}{n+1}\Psi^{n+2} . \tag{27}$$

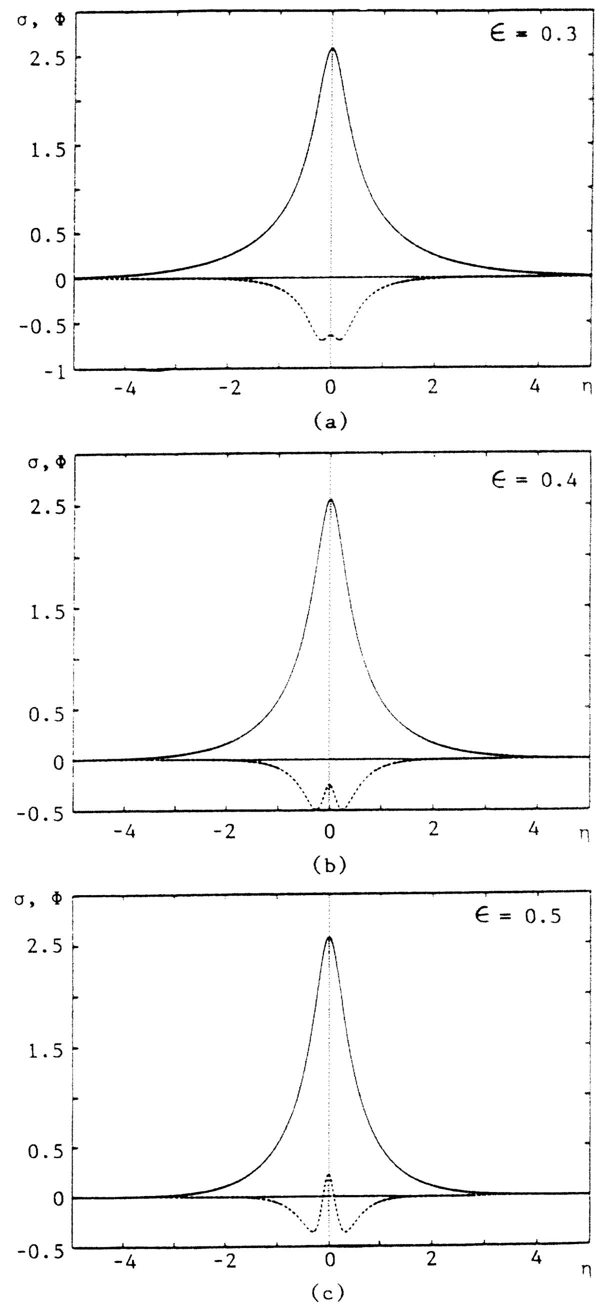

FIG. 1. Electromagnetic and bipolar fields as a function of η with fixed values of R, Ω and with different values of ϵ. (a) $R=0.12$, $\Omega=10$, $\epsilon=0.3$; (b) $\epsilon=0.4$; (c) $\epsilon=0.5$. (Solid line indicates σ, while dashed line indicates Φ.)

Now expressing b_n and a_n through the coefficients c_n and limiting with $n=1, 2$, finally we find

$$\left[V_0\left[\frac{d\Psi}{d\eta}\right]\right]^2=\alpha_1\Psi^2+\alpha_2\Psi^3+\alpha_3\Psi^4 , \tag{28}$$

with

$$\alpha_1=\frac{4(1-R)}{R}(1+\epsilon-\Omega R) ,$$
$$\alpha_2=1+\epsilon-\frac{2(1-\epsilon)}{R}c_1 , \tag{29}$$
$$\alpha_3=-\frac{2}{3R}\{(1+\epsilon)c_1^2+2(1-\epsilon)c_2\} .$$

For $\alpha_1>0$, Eq. (28) yields

$$\Psi=\frac{\beta_1\beta_2\mathrm{sech}^2(\chi\eta)}{\beta_2-\beta_1\tanh^2(\chi\eta)} , \tag{30}$$

where

$$\chi=\alpha_1^{1/2}/2V_0 ,$$
$$\beta_{1,2}=\frac{1}{2\alpha_3}[-\alpha_2\mp(\alpha_2^2-4\alpha_1\alpha_3)^{1/2}] . \tag{31}$$

Thus we conclude that the relativistically intense EM radiation may be localized in an admixture of unmagnetized electron-positron-ion plasma, which is not possible in the pure electron-positron plasma (i.e., when $\epsilon=1$) [13]. These localized fields (solitons) induce the bipolar potentials, which are moving with the velocity $V\approx c$ ($R\ll 1$). As discussed in Ref. [14], such a potential may accelerate resonant particles (electrons or positrons). The maximal increase of particle energy is $\Delta E\sim(mc^2\Phi_{\max}/R)$. (Note that in our case $\Phi_{\max}<1$.)

Here we express the generated potential as some function of the driving field intensity and use a series expansion. Some arguments for the convergence of the series are given in the Appendix, and the convergent solutions are shown in Fig. 1. It shows that the driving field intensity creates an intense soliton in the plasma with the generation of double hump bipolar potentials. With the increase of the value of ϵ, we find the tendency of a single hump soliton to converge to a double hump one. Note that our procedure is valid only for some regions of parameters (see Appendix). For $\Omega=10$ and $R=0.12$, ϵ has to be in the range $0.2<\epsilon<0.7$.

Consideration of ion dynamics, of course, may affect the localization phenomenon. This effect and the multidimensional case will be studied in our further investigations.

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APPENDIX

Here we give the explicit expressions for the derived polynomial of Eq. (22), the coefficients a_n, b_n, and c_n, and we discuss their convergence. Using the expansions (23) and (24) in (22), we get

$$\begin{aligned}
&-4\Omega\sum_{n=1}^{\infty}n(n-1)c_n\Psi^n-\frac{4}{R^2}\sum_{n=1}^{\infty}[b_n+a_n-2(1-\epsilon)c_n]\Psi^{n+1}\sum_{n=1}^{\infty}n(n-1)c_n\Psi^{n-2}\\
&-\frac{8}{R}\left[\frac{1}{R}(1+\epsilon)-\Omega\right]\Psi^2\left[\sum_{n=1}^{\infty}nc_n\Psi^{n-1}\right]^3-\frac{4}{V_0^2R^2}\sum_{n=1}^{\infty}(b_n+\epsilon a_n)\Psi^{n+2}\left[\sum_{n=1}^{\infty}nc_n\Psi^{n-1}\right]^3\\
&+\frac{4}{R^2}\left[-\frac{1}{2V_0^2}\sum_{n=1}^{\infty}(b_n-\epsilon a_n)\Psi^{n+1}+(1+\epsilon)\sum_{n=1}^{\infty}c_n\Psi^{n+1}-\frac{1}{2V_0^2}\sum_{n=1}^{\infty}c_n\Psi^{n+1}\sum_{n=1}^{\infty}(b_n+\epsilon a_n)\Psi^n\right]\\
&\times\left[\sum_{n=1}^{\infty}nc_n\Psi^{n-1}\right]^2+2\left\{\left[\frac{1}{R}(1+\epsilon)-2\Omega\right]\Psi-\frac{1}{R^2}\sum_{n=1}^{\infty}[b_n+a_n-2(1-\epsilon)c_n]\Psi^n\right.\\
&\qquad\left.-\frac{1}{2V_0^2R}\sum_{n=1}^{\infty}(b_n+\epsilon a_n)\Psi^{n+1}\right\}\sum_{n=1}^{\infty}nc_n\Psi^{n-1}\\
&\qquad-\frac{1}{R}\left\{(1+\epsilon)\sum_{n=1}^{\infty}c_n\Psi^n-\frac{1}{2V_0^2}\left[\sum_{n=1}^{\infty}(b_n-\epsilon a_n)\Psi^n+\sum_{n=1}^{\infty}c_n\Psi^n\sum_{n=1}^{\infty}(b_n+\epsilon a_n)\Psi^n\right]\right\}=0 .
\end{aligned} \tag{A1}$$

Inserting the first expression of Eq. (23) into the second and third, the coefficients a_n, b_n are determined in terms of the c_n's:

$$\begin{aligned} &a_1=-(2c_1+R)\ , \quad b_1=2c_1-R\ ,\\ &a_2=c_1^2-2c_2\ , \quad b_2=c_1^2+2c_2\\ &a_3=2(c_1c_2-c_3)\ , \quad b_3=2(c_3+c_1c_2)\\ &\quad\vdots \qquad\qquad\qquad \vdots \end{aligned} \tag{A2}$$

Then using (A2) in (A1) and equating the coefficients of the power series in Ψ, we get

$$c_1=\frac{R}{8(1-\epsilon)}\left\{4(\Omega R+1)-\frac{2-R}{1-R}(1+\epsilon)-\left\{\left[4(\Omega R+1)-\frac{2-R}{1-R}(1+\epsilon)\right]^2+\frac{8(1-\epsilon)^2}{R^2(1-R)}\right\}^{1/2}\right\}, \tag{A3}$$

$$c_2=-c_1[f_1/f_2]\ ,$$

where

$$f_1=\frac{1}{R^2}\left[(1+\epsilon)\left[-4-\frac{4}{V_0^2}\right]+8\Omega R-4\right]c_1^2+\left\{\frac{1}{R}(1-\epsilon)\left[-4-\frac{1}{2V_0^2}\right]\right\}c_1+\frac{1+\epsilon}{2V_0^2}\ , \tag{A4a}$$

$$f_2=-16\Omega+R^3+\frac{8}{R}+(1-\epsilon)\left[R^2+\frac{12}{R^2}\right]c_1+(1+\epsilon)\frac{1}{R}\left[3+\frac{1}{1-R}\right]\ . \tag{A4b}$$

We study the region of the free parameters, where the series (23) converges. From (30) and (31) we see that for the localized solution α_1, should be positive. Then for $0<R<1$, it yields the following condition:

$$0<R<\frac{(1+\epsilon)}{\Omega}\ . \tag{A5}$$

So with the fixed Ω and ϵ, R is the only free parameter of the system. One can find some range for the value of R with the above condition (A5) which yields

$$|c_1\Psi_{\max}+c_2\Psi_{\max}^2|<1\ , \tag{A6}$$

and

$$\begin{aligned} &c_3\Psi_{\max}^3\sim R\ ,\\ &c_4\Psi_{\max}^4\sim R^2\\ &\vdots \end{aligned} \tag{A7}$$

So for $R\to 0$, the series (23) converges rapidly with the chosen parameters as shown in Fig. 1. Although it is not a formal proof this argument supports the validity of the analytic development of our calculations. It is to be noted that, for the case when $|(1-\epsilon)|\ll 1$, the problem is to be treated in some other way as the series expansion is not valid in this case.

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